Data Representation

Learning Outcome

<u>Data Representation</u>
Represent data in binary and hexadecimal forms
Write programs to perform the conversion of positive integers between different number bases: denary, binary and hexadecimal forms; and display results
<u>Data Validation</u>
Understand data validation technique: check digit

1 Binary Representation

Computers only 'understand' **0**'s and **1**'s. All the data inside a computer has to be represented by patterns of **1**'s and **0**'s.

e.g.

0	1	0	1	1	0	1	0

represents character 'Z' in ASCII code

0	0	0	0	0	0	1	1

represents number 3

- A **bit** is a binary digit. It can be a **1** or **0**.
- A **byte** is taken to represent 8 bits.

e.g. A 16-bit computer

memory location	contents
0	**1 1 0 0 0 1 0 1 0 0 0 1 0 1 0 1**
1	**0 1 0 1 1 0 0 1 0 1 0 0 1 0 1 1**

a bit

a byte (8 bits)

2 Number Systems

System	*Base*	*Digits Used*	*Used by*
Decimal (Denary)	10	0,1,2,3,4,5,6,7,8,9	humans
Binary	**2**	**0,1**	**computers**
Octal	8	0,1,2,3,4,5,6,7	Programmers as 'shorthands' for binary
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F	

A number **N** with base **r** is written as $\mathbf{N}_{\mathbf{r}}$

e.g. 101_2 (Binary) 71_8 (Octal) $19A_{16}$ (Hex) 6925 (Decimal)

Place Value: the value of any digit depends on its position in the number.

e.g. Decimal number **4957**

place value :	10^3	10^2	10^1	10^0
	4	**9**	**5**	**7**

e.g. Binary number **1101** $_2$

place value :	2^3	2^2	2^1	2^0
	1	**1**	**0**	**1**

The most significant digit (**MSD**) is the digit with the highest place value in the number.
The least significant digit (**LSD**) is the digit with the lowest place value in the number.

e.g. $1\ 0\ 0\ 1\ 0_2$ (MSD: 1, LSD: 0) , $4\ 9\ 7\ 5_{10}$ (MSD: 4, LSD: 5)

3 Conversion of a number from Decimal to another base

(Decimal to Others) Method :

- Divide the decimal number by the new base continuously and note the remainders until the quotient is zero.
- Arrange the remainders, with the first remainder as the least significant digit and the last remainder as the most significant digit.

Example Convert 79_{10} to binary. **79_{10} = 1001111_2**

		remainder	
2	79		
2	39	------ **1**	(LSD)
2	19	------ **1**	
2	9	------ **1**	
2	4	------ **1**	
2	2	------ **0**	
2	1	------ **0**	
stopping condition →	0	------ **1**	(MSD)

Example Convert 79_{10} to hexadecimal. $\mathbf{79_{10} = 4F_{16}}$

```
16 | 7 9          remainder
   ------
16 |   4   ------   F   ↑ (LSD)
   ------
stopping  →    0   ------   4   • (MSD)
condition
```

note : F_{16} is equivalent to 15_{10}

4 Conversion of a number from other base to Decimal

(Others to Decimal) Method: Multiply each digit by its place value and then sum up the products.

Example Convert 100_2 to decimal

place value: 2^2 2^1 2^0

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1 0 0

100_2

$= (1 * 2^2) + (0 * 2^1) + (0 * 2^0)$

$= 4_{10}$

$\mathbf{100_2 = 4_{10}}$

Example Convert $6D_{16}$ to decimal

place value: 16^1 16^0

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6 D

$6D_{16}$

$= (6 * 16^1) + (D * 16^0)$

$= (6 * 16^1) + (13 * 16^0)$

$= 109_{10}$

$\mathbf{6D_{16} = 109_{10}}$

5 Conversion of a number from Binary to Hexadecimal and vice versa

(Binary to Hexadecimal) Method:

- Starting from LSD, sector in group of four. Insert dummy 0's when there is not enough
- For each group of four, calculate its decimal value, and it is the digit for hexadecimal

Example Convert 1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$ to hexadecimal

By sectoring in group of four

0 0 **1 0**	**1 0 1 0**	**0 1 0 1**	**0 1 1 1**	(base 2)
2	A	5	7	(base 16)

1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$ = 2 A 5 7 $_{16}$

Example Convert 1 1 1 0 0 $_2$ to hexadecimal

By sectoring in group of four

0 0 0 **1**	**1 1 0 0**	(base 2)
1	C	(base 16)

1 1 1 0 0 $_2$ = 1 C $_{16}$

(Hexadecimal to Binary) Method: Convert each digit into the binary representation of four digit

Example Convert 2 A 5 7 $_{16}$ to binary

2	A	5	7	(base 16)
0 0 1 0	**1 0 1 0**	**0 1 0 1**	**0 1 1 1**	(base 2)

2 A 5 7 $_{16}$ = 0 0 1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$

6 (Optional) Conversion between octal and other bases

Although octal bases are excluded from the new syllabus, it is an interesting part to learn.

Conversion between octal and binary is similar to the conversion between hexadecimal and binary.

Example Convert 1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$ to octal

By sectoring in group of three

0 **1 0**	**1 0 1**	**0 0 1**	**0 1 0**	**1 1 1**	(base 2)
2	5	1	2	7	(base 8)

1 0 1 0 1 0 0 1 0 1 0 1 1 1 $_2$ = 2 5 1 2 7 $_8$

Example Convert 7 0 1 5 $_8$ to binary

7	0	1	5	(base 8)
1 1 1	**0 0 0**	**0 0 1**	**1 0 1**	(base 2)

7 0 1 5 $_8$ = 1 1 1 0 0 0 0 0 1 1 0 1 $_2$

However, the conversion between octal and hexadecimal uses binary as an intermediate step.

Example Convert 2 A 5 7 $_{16}$ to octal

2	A	5	7	(base 16)
0 0 **0 0 1 0**	**1 0 1 0**	**0 1 0 1**	**0 1 1 1**	(base 2)

0 0 0	0 1 0	1 0 1	0 0 1	0 1 0	1 1 1	(base 2)
0	2	5	1	2	7	(base 8)

2 A 5 7 $_{16}$ = 2 5 1 2 7 $_8$

Tutorial

1. Perform the following conversion between different bases:

Binary 1010	Denary:
Denary 27	Binary:
Denary 63	Hexadecimal:
Hexadecimal B48	Denary:
Binary 10110	Hexadecimal:
Hexadecimal FF60	Binary:

2. For each conversion below, write a non-recursive function and a recursive function. Design your function with the data type of integer for decimal numbers, and string for binary and hexadecimal numbers.

   (a) Decimal numbers to binary numbers
   (b) Decimal numbers to hexadecimal numbers
   (c) Binary numbers to decimal numbers
   (d) Hexadecimal numbers to decimal numbers
   (e) Binary numbers to hexadecimal numbers
   (f) Hexadecimal numbers to binary numbers